Investigation • Points and Slope

Name ______________________ **Period** ________ **Date** __________

Hector recently signed up with a limited-usage Internet provider. There is a flat monthly charge and an hourly rate for the number of hours he is connected during the month. The table shows the amount of time he spent using the Internet for the first three months and the total fee he was charged.

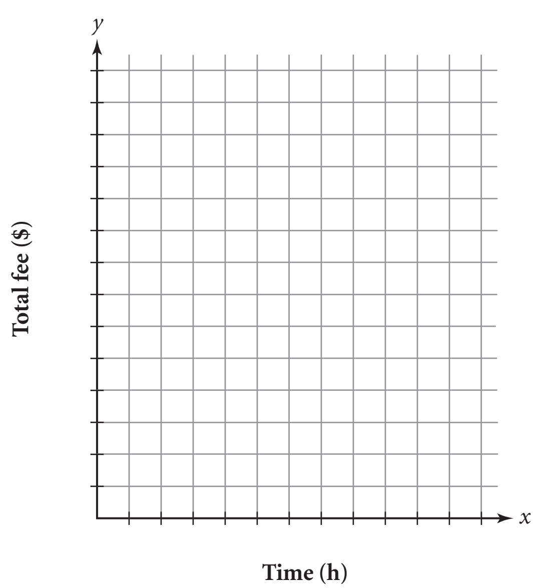
Step 1 Is there a linear relationship between the time in hours that Hector uses the Internet and his total fee in dollars? If so, why do you think such a relationship exists?

Internet Use

Month	Time (h)	Total fee ($)
September	40	16.55
October	50	19.45
November	80	28.15

Step 2 Use the numbers in the table to find the hourly rate in dollars per hour. Explain how you calculated this rate.

Step 3 Plot and label the three points the table of data represents. Draw a line through the three points. Does this line support your answer in Step 1?



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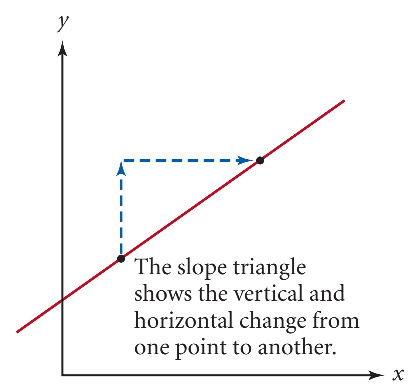
Step 4 Choose two points on your graph. Use arrows to show how you could move from one point to the other using only one vertical move and one horizontal move. How long is each arrow? What are the units of these values?

Step 5 How do the arrow lengths relate to the hourly rate that you found in Step 2? Use the arrow lengths to find the hourly rate of change, or slope, for this situation. What units should you apply to the number?

In Step 4, you used arrows to show the vertical change and the horizontal change when you moved from one point to another. The right triangle you created is called a **slope triangle.**

Step 6 Choose a different pair of points on your graph. Create a slope triangle between them and use it to find the slope of the line. How does this slope compare to your answers in Step 2 and Step 5?



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Step 7 Think about what you have done with your slope triangles. How could you use the coordinates of any two points to find the vertical change and the horizontal change of each arrow? Write a single numerical expression using the coordinates of two points to show how you can calculate slope.

Step 8 Write a symbolic algebraic rule for finding the slope between any two points (x_1, y_1) and (x_2, y_2). The subscripts mean that these are two distinct points of the form (x, y).

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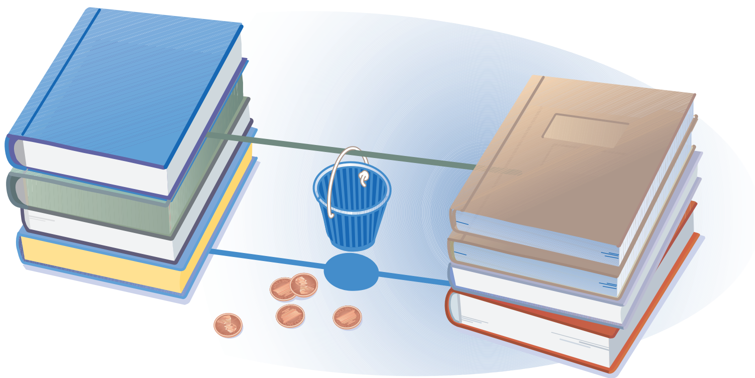
Investigation • Beam Strength

Name ______________________ **Period** __________ **Date** ____________

You will need: graph paper, uncooked spaghetti, several books, a plastic cup, string, pennies

How strong do the beams in a ceiling have to be? How do bridge engineers select beams to support traffic? In this investigation you will collect data and find a linear model to determine the strength of various "beams" made of spaghetti.



Step 1 Make two stacks of books of equal height. Punch holes on opposite sides of the cup and tie the string through the holes.

Step 2 Follow the Procedure Note for a beam made from one strand of spaghetti. Record the maximum load (the number of pennies) that this beam will support in the table.

Procedure Note

1. Hang your cup at the center of your spaghetti beam.
2. Support the beam between the stacks of books so that it overlaps each stack by about 1 inch. Put another book on each stack to hold the beam in place.
3. Put pennies in the cup, one at a time, until the beam breaks.

Number of strands	Number of pennies
1	
2	
3	
4	
5	
6	

Step 3 Repeat Step 2 for beams made from two, three, four, five, and six strands of spaghetti. Record the data in the table.

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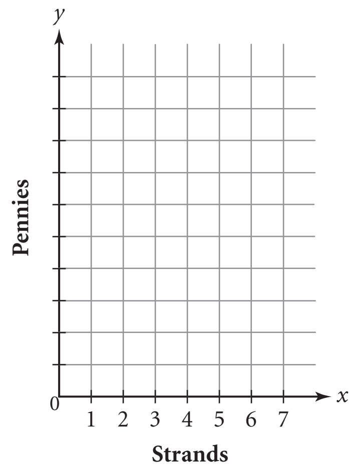
Step 4 Plot your data on your calculator. Let x represent the number of strands of spaghetti, and let y represent the maximum load. Sketch the plot on this grid too.



Step 5 Use a strand of spaghetti to visualize a line that you think fits the data on your sketch. Choose two points on the line. Note the coordinates of these points. Calculate the slope of the line between the two points.

Step 6 Use the slope, b, that you found in Step 5 to graph the equation $y = bx$ on your calculator. Why is this line parallel to the direction the points indicate? Is the line too low or too high to fit the data?

Step 7 Using the spaghetti strand on your sketch, estimate a good y-intercept, a, so that the equation $y = a + bx$ better fits your data. On your calculator, graph the equation $y = a + bx$ in place of $y = bx$. Adjust your estimate for a until you have a line of fit.

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Step 8 In Step 5, everyone started with a visual model that went through two points. In your group, compare all final lines. Did everyone end up with the same line? Do you think a line of fit must go through at least two data points? Is any one line better than the others?

Your line is a model for the relationship between the number of strands of spaghetti in the beam and the load in pennies that the beam can support.

Step 9 Explain the real-world meaning of the slope of your line.

Step 10 Use your linear model to predict the number of spaghetti strands needed to support \$5 worth of pennies.

Step 11 Use your model to predict the maximum loads for beams made of 10 and 17 strands of spaghetti.

Step 12 Some of your data points may be very close to your line, while others could be described as outliers. What could have caused these outliers?

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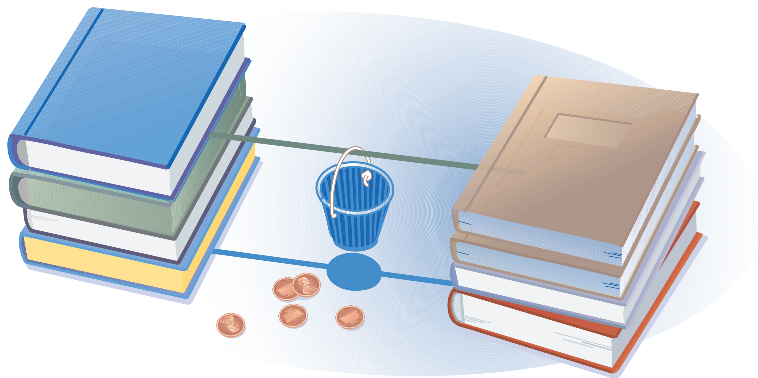
Name ______________________ **Period** __________ **Date** ____________

How strong do the beams in a ceiling have to be? How do bridge engineers select beams to support traffic? In this investigation you will use data and find a linear model to determine the strength of various "beams" made of spaghetti.



Step 1 The beam strength experiment was set up as shown in the picture.

Step 2 The Procedure Note was followed for a beam made from one strand of spaghetti. The maximum load (the number of pennies) that this beam will support was recorded in the table.

Step 3 Step 2 was repeated for beams made from two, three, four, five, and six strands of spaghetti. The table shows the data collected by one group of students.

Number of strands	Number of pennies
1	10
2	16
3	28
4	34
5	41
6	46

Procedure Note

1. Hang your cup at the center of your spaghetti beam.
2. Support the beam between the stacks of books so that it overlaps each stack by about 1 inch. Put another book on each stack to hold the beam in place.
3. Put pennies in the cup, one at a time, until the beam breaks.

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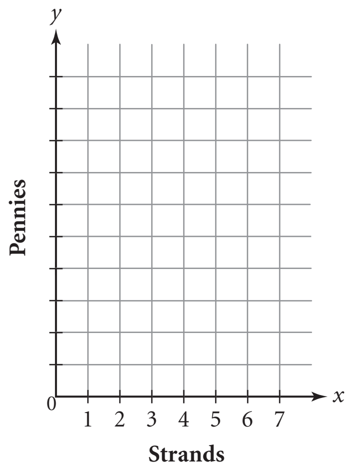
Step 4 Plot the data on your calculator. Let x represent the number of strands of spaghetti, and let y represent the maximum load. Sketch the plot on this grid too.



Step 5 Use the edge of a ruler or piece of paper to visualize a line that you think fits the data on your sketch. Choose two points on the line. Note the coordinates of these points. Calculate the slope of the line between the two points.

Step 6 Use the slope, b, that you found in Step 5 to graph the equation $y = bx$ on your calculator. Why is this line parallel to the direction the points indicate? Is the line too low or too high to fit the data?

Step 7 Using the edge of a ruler or piece of paper on your sketch, estimate a good y-intercept, a, so that the equation $y = a + bx$ better fits your data. On your calculator, graph the equation $y = a + bx$ in place of $y = bx$. Adjust your estimate for a until you have a line of fit.

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Step 8 In Step 5, everyone started with a visual model that went through two points. In your group, compare all final lines. Did everyone end up with the same line? Do you think a line of fit must go through at least two data points? Is any one line better than the others?

Your line is a model for the relationship between the number of strands of spaghetti in the beam and the load in pennies that the beam can support.

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Step 10 Use your linear model to predict the number of spaghetti strands needed to support $5 worth of pennies.

Step 11 Use your model to predict the maximum loads for beams made of 10 and 17 strands of spaghetti.

Step 12 Some of your data points may be very close to your line, while others could be described as outliers. What could have caused these outliers?

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Investigation • The Point-Slope Form for Linear Equations

Name ______________________________ **Period** __________ **Date** ______________

Silo and Jenny conducted an experiment in which Jenny walked at a constant rate. Unfortunately, Silo recorded only the data shown in this table.

Elapsed time (s) x	Distance to walker (m) y
3	4.6
6	2.8

Step 1 Find the slope of the line that represents this situation.

Step 2 Write a linear equation in point-slope form using the point (3, 4.6) and the slope you found in Step 1.

Step 3 Write another linear equation in point-slope form using the point (6, 2.8) and the slope you found in Step 1.

Step 4 Enter the equation from Step 2 into Y1 and the equation from Step 3 into Y2 on your calculator, and graph both equations. What do you notice?

Step 5 Look at a table of Y1- and Y2-values. What do you notice? What do you think the results mean?

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Now that you have some practice at writing point-slope equations, try using a point-slope equation to fit data.

The table shows how the temperature of a pot of water changed over time as it was heated.

Water Temperature

Time (s) x	Temperature (°C) y
24	25
36	30
49	35
62	40
76	45
89	50

Step 6 Define variables and plot the data on your calculator. Describe any patterns you notice.

Step 7 Choose a pair of points from the data. Find the slope of the line between your two points.

Step 8 Write an equation in point-slope form for a line that passes through your two points. Graph the line on your calculator. Does your equation fit the data?

Step 9 Compare your graph to those of other members of your group. Does one graph show a line that is a better fit than the others? Explain.

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Investigation • Equivalent Equations

Name ______________________ **Period** __________ **Date** ____________

Here are six different-looking equations in point-slope form.

a. $y = 3 - 2(x - 1)$ **b.** $y = -5 - 2(x - 5)$ **c.** $y = 9 - 2(x + 2)$

d. $y = 0 - 2(x - 2.5)$ **e.** $y = 7 - 2(x + 1)$ **f.** $y = -9 - 2(x - 7)$

Step 1 Do the six equations represent the same line or different lines? Explain.

Step 2 Divide these equations among the members of your group. Use the distributive property to rewrite the right side of each equation. When you combine like terms, you should get an equation in intercept form.

Step 3 Enter your point-slope equation into Y_1, and enter your intercept equation into Y_2. Check that the two equations have the same calculator graph or table. How does this show that the equations are equivalent?

Step 4 Now, as a group, compare your intercept equations. What do the results show about the six equations?

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Investigation • Equivalent Equations (continued)

Step 5 As a group, explain how you can tell that an equation in point-slope form is equivalent to one in intercept form. Think about how you can do this graphically and symbolically.

Here are fifteen equations. They represent only four different lines.

Step 6 Test your answer to Step 5 by finding the intercept form of each equation and then grouping equivalent equations.

a. $y = 2(x - 2.5)$ **b.** $y = 18 + 2(x - 8)$

c. $y = 52 - 6(x + 8)$ **d.** $y = -6 + 2(x + 4)$

e. $y = 21 - 6(x + 4)$ **f.** $y = -14 - 6(x - 3)$

g. $y = -10 + 2(x + 6)$ **h.** $6x + y = 4$

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i. $y = 11 + 2(x - 8)$

j. $12x + 2y = -6$

k. $y = 2(x - 4) + 10$

l. $y = 15 - 2(10 - x)$

m. $y = 7 + 2(x - 6)$

n. $y = -6(x + 0.5)$

o. $y = -6(x + 2) + 16$

Step 7 As a group, explain how you can tell that two equations in point-slope form are equivalent.

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Investigation • Life Expectancy

Name ______________________ **Period** __________ **Date** ____________

You will need: graph paper (optional)

This table shows the relationship between the number of years a person might be expected to live and the year he or she was born. Life expectancy is a prediction that is very useful in professions like medicine and insurance.

Step 1 Choose one column of life expectancy data—female, male, or combined. Let x represent birth year, and let y represent life expectancy in years. Graph the data points on your calculator.

Step 2 Choose two points on your graph so that a line through them closely reflects the pattern of all the points on the graph. Use the two points to write the equation of this line in point-slope form.

U.S. Life Expectancy at Birth

Birth year	Female	Male	Combined
1940	65.2	60.8	62.9
1950	71.1	65.6	68.2
1960	73.1	66.6	69.7
1970	74.7	67.1	70.8
1975	76.6	68.8	72.6
1980	77.5	70.0	73.7
1985	78.2	71.2	74.7
1990	78.8	71.8	75.4
1995	78.9	72.5	75.8
2000	79.5	74.1	76.9

(National Center for Health Statistics, in *The World Almanac and Book of Facts 2004*, p.76)
[Data sets: **LEYR, LEFEM, LEMAL, LECOM**]

Step 3 Graph the line with your data points. Does it fit the data?

Step 4 Use your equation to predict the life expectancy of a person who will be born in 2022.

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Step 5 Compare your prediction from Step 4 to the prediction that another group made analyzing the same data. Are your predictions the same? Are they close? Explain why it's possible to make different predictions from the same data.

Step 6 Compare the slope of your line of fit to the slopes that other groups found working with different data sets. What does the slope for each data set tell you?

Step 7 As a class, select one line of fit that you think is the best model for each column of data—female, male, and combined. Graph all three lines on the same set of axes on your calculator. Is it reasonable for the line representing the combined data to lie between the other two lines? Explain why or why not.

Step 8 How does the point-slope method of finding a line compare to the intercept-form method? What are the strengths and weaknesses of each method?

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Investigation • Bucket Brigade

Name ______________________ **Period** __________ **Date** ____________

You will need: a stopwatch, a bucket

In this investigation you will use a systematic method for finding a particular line of fit for data.

Step 1 Line up in a bucket brigade. (See the Procedure Note.) Record the number of people in the line. Starting at one end of the line, pass the bucket as quickly as you can to the other end. Record the total passing time from picking up the bucket to setting it down at the very end in the table.

Procedure Note

Select a class member as timer. Everyone should line up single file. Your line might wrap around the room. Spread out so that there is an arm's length between two people.

Number of people	Passing time (s)

Step 2 Now have one or two people sit down and close up the gaps in the line. Repeat the bucket passing. Record the new number of people and the new passing time.

Step 3 Continue the bucket brigade until you have collected 10 data points in the form (*number of people, passing time in seconds*).

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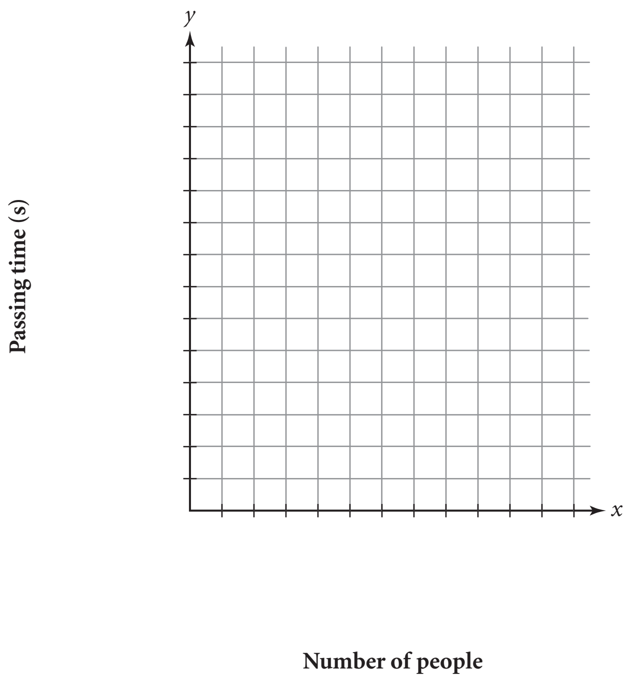
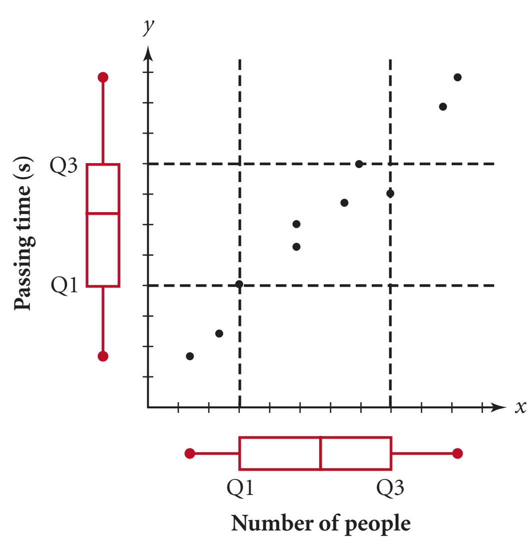
Step 4 Let x represent the number of people, and let y represent time in seconds. Plot your data on this graph.



Step 5 List the five-number summary for the x-values and the five-number summary for the y-values.

Step 6 What are the first-quartile (Q1) and third-quartile (Q3) values for the x-values in your data set? What are the Q1- and Q3-values for the y-values in your data set?

Step 7 On your graph, draw a horizontal box plot just below the x-axis using the five-number summary for the x-values. Draw a vertical box plot next to the y-axis using the five-number summary for the y-values. A sample graph is shown. Your data and graph will look different based on the data that you collect.



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Step 8 Draw vertical lines from the Q1- and Q3-values on the x-axis box plot into the graph. Draw horizontal lines from the Q1- and Q3-values on the y-axis box plot into the graph. These lines should form a rectangle in the plot. The vertices of this rectangle are called **Q-points.** Do the Q-points have to be actual data points? Why or why not? Will everyone get the same Q-points?

Step 9 Draw the diagonal of this rectangle that shows the direction of the data. Extend this diagonal through the plot. Is the line a good fit for the data? Are any of the original data points on your line? If so, which ones?

Step 10 Find the coordinates of the two Q-points the line goes through and write a point-slope equation for the line.

Step 11 What are the real-world meanings of the slope and y-intercept of this model?

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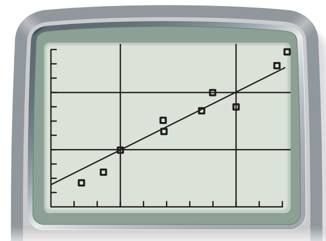
Step 12 What are the advantages and disadvantages of having a systematic procedure for finding a model for data?

Step 13 Use your calculator to plot the data points, draw the vertical and horizontal lines, and plot a line of fit found by this method. [▶ See **Calculator Note 4B: Using the DRAW Menu** for help on using the draw menu.◀]



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Investigation • Bucket Brigade

With Sample Data

Name ______________________ **Period** __________ **Date** ______________

In this investigation you will use a systematic method for finding a particular line of fit for data.

Step 1 A group of students lined up to perform the experiment. Starting at one end of the line, a bucket was quickly passed to the other end. The number of people in the line and the total passing time, from picking up the bucket to setting it down at the end, was recorded in a table.

Step 2 A few people sat down (one person in this case) and the students closed up the gap in the line. The bucket was passed again, and the new number of people and passing time were recorded in the table.

Step 3 The bucket brigade continued until data from 10 trials was recorded in the table.

Procedure Note

Select a class member as timer. Everyone should line up single file. Your line might wrap around the room. Spread out so that there is an arm's length between two people.

Number of people	Passing time (s)
22	16
21	18
18	12
16	14
15	12
13	11
12	9
11	7
8	8
5	4

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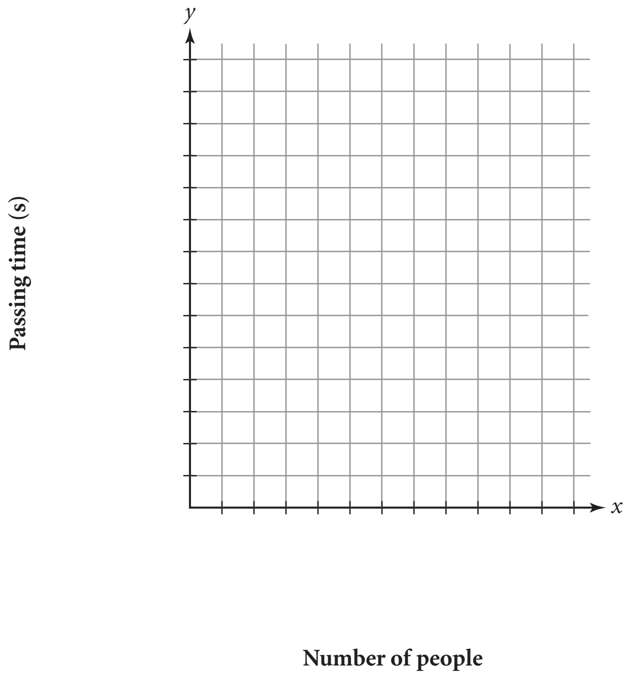
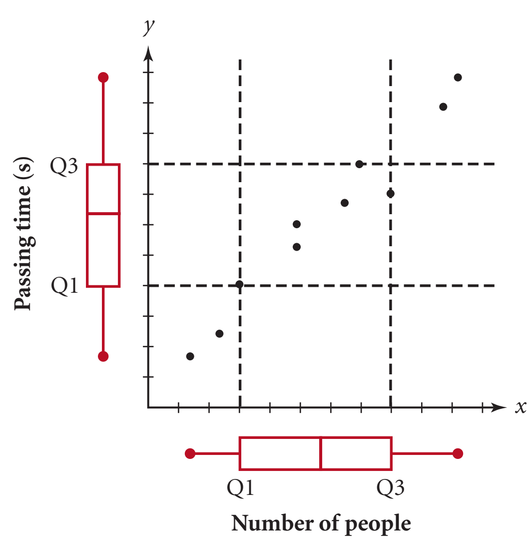
Step 4 Let x represent the number of people, and let y represent time in seconds. Plot your data on this graph.



Step 5 List the five-number summary for the x-values and the five-number summary for the y-values.

Step 6 What are the first-quartile (Q1) and third-quartile (Q3) values for the x-values in your data set? What are the Q1- and Q3-values for the y-values in your data set?

Step 7 On your graph, draw a horizontal box plot just below the x-axis using the five-number summary for the x-values. Draw a vertical box plot next to the y-axis using the five-number summary for the y-values. A sample graph is shown. Your data and graph will look different based on the data that you collect.



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Step 8 Draw vertical lines from the Q1- and Q3-values on the *x*-axis box plot into the graph. Draw horizontal lines from the Q1- and Q3-values on the *y*-axis box plot into the graph. These lines should form a rectangle in the plot. The vertices of this rectangle are called **Q-points.** Do the Q-points have to be actual data points? Why or why not? Will everyone get the same Q-points?

Step 9 Draw the diagonal of this rectangle that shows the direction of the data. Extend this diagonal through the plot. Is the line a good fit for the data? Are any of the original data points on your line? If so, which ones?

Step 10 Find the coordinates of the two Q-points the line goes through and write a point-slope equation for the line.

Step 11 What are the real-world meanings of the slope and *y*-intercept of this model?

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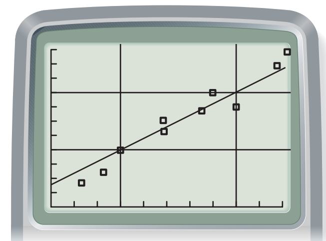
Step 12 What are the advantages and disadvantages of having a systematic procedure for finding a model for data?

Step 13 Use your calculator to plot the data points, draw the vertical and horizontal lines, and plot a line of fit found by this method. [▶ See **Calculator Note 4B: Using the DRAW Menu** for help on using the draw menu.◀]



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Investigation • What's My Line?

Name ______________________________ **Period** __________ **Date** ______________

You will need: a strand of spaghetti

This table shows data that Edwin Hubble used in 1929 to formulate Hubble's Law. The table includes the distance from Earth to known nebulae (clouds of gas or dust) measured in megaparsecs (Mpc, about 3.258×10^6 light-years), and the speed at which each nebula is moving away from or toward Earth. In this investigation, you'll create three linear models, analyze your models, and use them to make a prediction.

Distance and Speed of Nebulae

Distance (Mpc)	Speed (km/s)	Distance (Mpc)	Speed (km/s)
0.032	170	0.9	650
0.034	290	0.9	150
0.214	−130	0.9	500
0.263	−70	1.0	920
0.275	−185	1.1	450
0.275	−220	1.1	500
0.45	200	1.4	500
0.5	290	1.7	960
0.5	270	2.0	500
0.63	200	2.0	850
0.8	300	2.0	800
0.9	−30	2.0	1090

(Edwin Hubble, in *Proceedings of the National Academy of Sciences,* Volume 15, Number 3) [Data sets: **GXYDS, GXYSP**]

First, you'll find a line of fit using an "eyeballing" method. Remember that the object of a linear model is to summarize or generalize the data.

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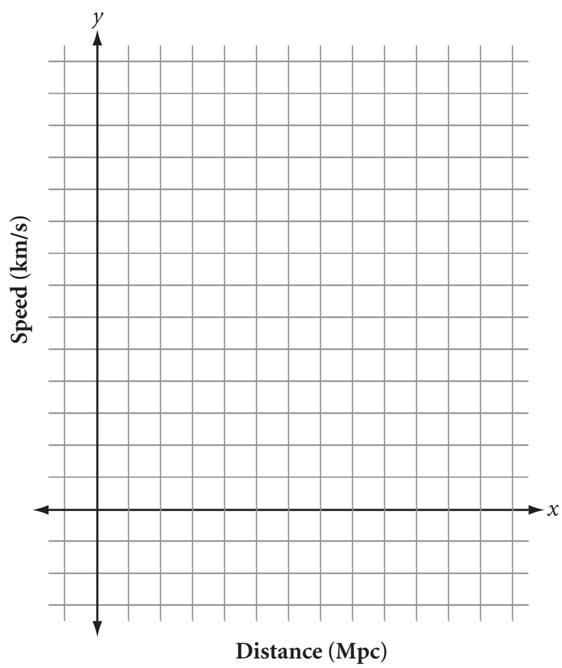
Investigation • What's My Line? (continued)

Step 1 Plot the data on the graph. Lay a piece of spaghetti on the plot so that it crosses the y-axis and follows the direction of the data. Try to focus not on the points themselves, but on the general direction of the "cloud" of points.



Step 2 Estimate the y-intercept. Locate a point with convenient coordinates along the strand. Use this information to write the equation of the line.

Next, you'll find a line of fit by choosing "representative" data points.

Step 3 Make a scatter plot of the data on your calculator. Choose two data points that you think show the direction of the data.

Step 4 Use the two points to write a linear equation in point-slope form.

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Next, you'll find a line of fit using Q-points.

Step 5 Use your calculator to get the five-number summaries for the x- and y-values. On the graph from Step 1 draw a rectangle using the first- and third-quartile values for the x-values and the first- and third-quartile values for the y-values. Name the Q-points you should use for the data.

Step 6 Write the equation of the line of fit you can draw through your selected Q-points. Graph that equation on the graph used in Step 1 to verify that is the diagonal of the rectangle you drew in Step 5.

Finally, you'll compare the lines and their characteristics and decide which method has given you the best-fitting line.

Step 7 Compare the slopes of all three lines from Steps 2, 4 and 6. Do all these numbers have the same real-world meaning? If so, what is it?

Step 8 Compare the y-intercepts of all three lines. Do they all have the same real-world meaning? If so, what is it?

Step 9 What distance would you expect for a galaxy that is moving away from Earth at a rate of 750 km/s? Show how to find this value symbolically.

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Step 10 What is the effect of a small change in the *y*-intercept when you use the model to predict a value in the middle of the data set?

Step 11 What is the effect of a small change in the slope when you try to predict a *y*-value far from most of the given points?

Step 12 Discuss the pros and cons of each procedure you used to find a line of fit. Which method do you like best and why?

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